

Assignment 2

Due Thursday, September 10

Work these on your own paper. Show enough of the reasoning that a reader could follow it without you in the room.

1. Finding solutions

Solve each initial-value problem by separating variables or undoing the product rule. You choose which approach to use!

For each solution, state its domain: the largest interval containing the initial time on which it solves the differential equation. (It may be all of $\mathbb{R}$! Or the solution may blow up at a finite time.)

(a) $y' + 2y = e^t, \quad y(0) = 1.$

(b) $y' = 2t(1 + y^2), \quad y(0) = 0.$

(c) $ty' + 2y = t^2, \quad y(1) = 0.$

(d) $y' = e^t(1 - y), \quad y(0) = 2.$

(e) $y' + 2ty = 2t^3, \quad y(0) = 1.$

(f) $y' = te^{-y}, \quad y(0) = 0.$

2. Find the combination

Consider

$$y' = t^2 + 2ty + y^2 - 3t - 3y + 1.$$

Neither separating variables nor undoing the product rule seems to apply as written. But a combination of algebra and calculus will let us succeed.

- (a)** Group the terms on the right-hand side to reveal the repeated combination $t + y$. Set $u = t + y$, and derive a differential equation for u involving only u and its derivative. What can we now do that we could not do before?
- (b)** Solve the original equation with initial condition $y(0) = 0$ by first solving for u and then returning to y . State the domain of your solution.
- (c)** Find the constant solutions of the u -equation. Translate each into a solution of the original equation, and verify both by substitution.